Chapter 2 (Part 1): Ordinal Games in Strategic Form

ECON 5001
Prof. Healy
Based on: *Game Theory* by Giacomo Bonanno

Updated 2026-08-27 at 01:36:34

First: A Video

Click me: The Golden Balls Video

https://www.facebook.com/SebTalksBoxing/videos/woman-wins-100k-with-sensational-shthousery/1127818344236313/

Golden Balls

A British game show. Two contestants, Sarah and Steven, have built up a jackpot of $100,000.

Each secretly picks one of two balls:

- one says **Split**
- one says **Steal**

They choose **simultaneously**. Then both are revealed.

- Both Split: each gets $50,000
- One Steals: the stealer gets all $100,000, the other gets nothing
- Both Steal: nobody gets anything

Golden Balls: The Table

		Steven	
		Split	Steal
Sarah	Split	Sarah: \$50k Steven: \$50k	Sarah: \$0 Steven: \$100k
	Steal	Sarah: \$100k Steven: \$0	Sarah: \$0 Steven: \$0

Each row is a choice for Sarah, each column a choice for Steven.
Each cell is what **happens** (the “outcome”)

This Game Has a Name

The most studied game in game theory:

Prisoner's Dilemma

Two players, each with a **cooperate** option and a **defect** option, where

- defecting looks better *no matter what* the other player does, and
- if both defect, both end up worse off than if both had cooperated

Split = cooperate. Steal = defect.

The Same Dilemma, Out in the Wild

Messier stories, same skeleton (**cooperate** vs. **defect**):

- **Arms races**: hold your build-up vs. out-arm them — both spend fortunes, neither safer
- **Carbon emissions**: cut vs. free-ride on everyone else's cuts
- **A shared fishery**: fish your quota vs. overfish — the stock collapses for all
- **Doping in sport**: race clean vs. dope — if everyone dopes, the medals go to the same people, at a cost
- **Price wars**: hold your price vs. undercut the rival — margins vanish on both sides
- **Ad spending**: modest campaign vs. outspend the rival — market shares barely move
- **Antibiotics**: prescribe sparingly vs. prescribe on demand — resistance is everyone's problem
- **Attack ads**: stay positive vs. go negative — both get dragged down
- **Standing at a concert**: stay seated vs. stand for a better view — everyone stands, nobody sees more

Theory vs. Reality

The gaps between theory and reality:

- **many** players, not two (fishing fleets, countries, fans in a stadium)
- **degrees** of cooperation, not two buttons (how much to emit, how far to cut price)
- **repetition**, reputation, and the chance to retaliate later
- **imperfect monitoring**: you can't always tell who defected
- **outside enforcement**: treaties, regulators, drug testing, cartel policing

None of that changes the core tension:

what's best for me, taken alone, leaves us all worse off.

Before We Analyze It: Let's Play

- Phone, tablet, or laptop (whatever you've got)
- Log in to **MobLab** and join our session
- Class code: `dv1cf8ft`
- One decision: **Split** or **Steal** (Cooperate or Defect)

No talking, no signaling. Choose what you'd actually choose.

A Tempting Argument

		Steven	
		Split	Steal
Sarah	Split	$50k, $50k	$0, $100k
	Steal	$100k, $0	$0, $0

Sarah's amount listed first

Contingent reasoning

what to do in each case

Sarah can't control Steven. But she can consider both cases.

- Suppose Steven picks **Steal**:

Best response = any

- Suppose Steven picks **Split**:

Best Response = Steal

- Therefore Sarah should STEAL is always a best response

(SPLIT is not if Steven steals)

What's Wrong With That Argument?

The argument snuck in an assumption.

What’s Wrong With That Argument?

We assumed Sarah is **selfish and greedy**: she cares only about her own money, and more is better.

That might be false! Sarah might:

- value **fairness**: an even split beats grabbing everything
- feel **benevolence**: if she gets nothing anyway, she’d rather Steven get something
- feel **spite**: she’d rather Steven get nothing

Under some of these, the “obvious” answer flips completely.

Moral: the table above is not yet enough to say what’s rational.

Frames vs. Games

What the table gave us:

- the players
- what each can do
- what **outcome** results from each combination

What the table did *not* give us:

- how each player **ranks** those outcomes

$$\text{Table alone} = \textbf{game-frame}$$

$$\text{Frame} + \text{rankings} = \textbf{game}$$

Our Own Example: Sunday Morning

Long Saturday night. Alex and Blair both forgot to charge their phones, and both need a serious breakfast.
No way to message. Each just walks to one of the two places they always go.

- Alex chooses: Buckeye Donuts (B) or HangOverEasy (H)
- Blair chooses: Buckeye Donuts (B) or HangOverEasy (H)
- They choose simultaneously (neither sees the other's choice)

Four things can happen:

- o_1: both at Buckeye Donuts
- o_2: Alex at BuckyDs, Blair at HangOE (they eat alone)
- o_3: Alex at HangOE, Blair at BuckyDs (they eat alone)
- o_4: both at HangOverEasy

Sunday Morning as a Frame

- Players: $I = \{1, 2\}$ $(1 = \text{Alex}, 2 = \text{Blair})$
- Strategies: $S_1 = \{B, H\}$, $S_2 = \{B, H\}$
- Profiles: $S = S_1 \times S_2 = \{(B, B), (B, H), (H, B), (H, H)\}$
- Outcomes: $O = \{o_1, o_2, o_3, o_4\}$
- Outcome function: $f : S \to O$

s:	(B, B)	(B, H)	(H, B)	(H, H)
$f(s)$:	o_1	o_2	o_3	o_4

Notice: **nothing here says what anybody wants.**

Definition: Game-Frame in Strategic Form

A **game-frame in strategic form** is a list of four items

$$\langle I,\ (S_1, S_2, \ldots, S_n),\ O,\ f \rangle$$

where

- $I = \{1, 2, \ldots, n\}$ is the set of **players** ($n \geq 2$)
- S_i is the set of **strategies** available to player i
    - $S = S_1 \times S_2 \times \cdots \times S_n$ is the set of **strategy profiles**
    - a typical profile is $s = (s_1, \ldots, s_n)$
- O is the set of **outcomes**
- $f : S \to O$ is the **outcome function**

Why a function?

Golden Balls as a Frame

Write it out:

- $I =$
- $S_1 =$ $S_2 =$
- $O =$
- $f =$

Preferences

The Problem

Outcomes are *things*, not numbers:

- "both at Buckeye Donuts"
- "Sarah gets $100,000 and Steven gets nothing"
- "the park gets built and I pay $40"

We can't add them, average them, or take derivatives.

All we will assume a player can do is **compare** them, two at a time.

One Relation to Rule Them All

For each player i we write down a single relation on O:

$$o \succsim_i o'$$

"Player i considers o to be **at least as good as** o'"

This one relation is enough. From it we *define* the other two:

- **Strict preference:** $o \succ_i o'$ if $o \succsim_i o'$ and *not* $o' \succsim_i o$
- **Indifference:** $o \sim_i o'$ if $o \succsim_i o'$ and $o' \succsim_i o$

You've Done This Before

You already know a relation that works exactly this way: $\geq$ on numbers.

Numbers	Outcomes	
$x \geq y$	$o \succsim_i o'$	at least as good as
$x > y$	$o \succ_i o'$	strictly better than
$x = y$	$o \sim_i o'$	just as good as

And the definitions are the same moves you'd make in grade school:

- $x > y$ means $x \geq y$ and *not* $y \geq x$
- $x = y$ means $x \geq y$ and $y \geq x$

Where the Analogy Breaks

With numbers, "at least as big" comes for free:

- any two numbers *can* be compared
- the comparisons *never* cycle

With outcomes, neither is automatic. "Both at Buckeye Donuts" and "Sarah gets $100,000" are not on a number line.

So the two properties we get for free with $\geq$ become **assumptions** about $\succsim_i$.

That's next.

Reading the Symbols

Notation	Interpretation
$o \succsim_i o'$	i finds o at least as good as o' (better than, *or* just as good as)
$o \succ_i o'$	i finds o better than o'; i *prefers* o to o'
$o \sim_i o'$	i finds o just as good as o'; i is *indifferent*

The subscript i matters: preferences belong to **a person**.

Let C be dinner at Cazuela's and Y be dinner at Yoshi Sushi. Then

$$C \succ_{\text{Alice}} Y \quad \text{but} \quad C \sim_{\text{Bob}} Y$$

is perfectly consistent: Alice prefers Cazuela's, Bob can't tell the difference.

Chains are read left to right, best to worst:

$$o_3 \succ o_1 \succ o_2 \sim o_4$$

Two Assumptions We Will Always Make

1. **Completeness.** For any two outcomes o and o':

$$o \succsim_i o' \quad \text{or} \quad o' \succsim_i o \quad \text{(or both)}$$

- i.e., the player is never unable to compare
- “I don’t know” and “I don’t care” are different!

2. **Transitivity.** For any three outcomes:

$$\text{if } o_1 \succsim_i o_2 \;\text{ and }\; o_2 \succsim_i o_3 \;\text{ then }\; o_1 \succsim_i o_3$$

- i.e., the ranking doesn’t cycle

Why Transitivity?

Suppose your preferences *cycle*: $x \succ y, \; y \succ z, \;$ but $\; z \succ x$.

You own z. I own x and y.

- You prefer y to z, so you'd pay me a penny to trade z for y
- You prefer x to y, so you'd pay me a penny to trade y for x
- You prefer z to x, so you'd pay me a penny to trade x for z

Now you're back where you started, three cents poorer. Repeat forever.

This "money pump" argument is not in the text, but is standard.

Transitivity Buys Us a Lot

If $\succsim_i$ is complete and transitive, then $\succ_i$ and $\sim_i$ inherit good behavior:

- if $o_1 \sim o_2$ and $o_2 \sim o_3$ then $o_1 \sim o_3$
- if $o_1 \succ o_2$ and $o_2 \succ o_3$ then $o_1 \succ o_3$
- if $o_1 \succ o_2$ and $o_2 \sim o_3$ then $o_1 \succ o_3$
- if $o_1 \sim o_2$ and $o_2 \succ o_3$ then $o_1 \succ o_3$

Together these let us line the outcomes up from best to worst, with ties allowed.

Three Ways to Write the Same Ranking

Take $O = \{o_1, o_2, o_3, o_4, o_5\}$ and the ranking

$$o_3 \succ o_5 \sim o_1 \succ o_4 \succ o_2$$

Way 1: chain notation. Exactly what's written above.

Way 2: a list, best on top. Ties share a row.

$$\begin{array}{rl} \text{best} & o_3 \\ & o_1,\ o_5 \\ & o_4 \\ \text{worst} & o_2 \end{array}$$

Three Ways to Write the Same Ranking

Way 3: attach a number to each outcome.

outcome	o_1	o_2	o_3	o_4	o_5
U	6	1	8	2	6

Rule: bigger number = better; equal numbers = indifferent.

Check it against $o_3 \succ o_5 \sim o_1 \succ o_4 \succ o_2$:

Definition: Ordinal Utility Function

Let $\succsim$ be a complete and transitive ranking of a finite set O.

A function $U : O \rightarrow \mathbb{R}$ is an **ordinal utility function representing** $\succsim$ if, for all outcomes o and o':

$$U(o) > U(o') \text{ if and only if } o \succ o'$$

$$U(o) = U(o') \text{ if and only if } o \sim o'$$

Equivalently: $\quad o \succsim o' \text{ if and only if } U(o) \geq U(o').$

$U(o)$ is called the **utility** of outcome o.

Utility Numbers Are Arbitrary

All three of these represent the *same* ranking $o_3 \succ o_1 \succ o_2 \sim o_4$:

	o_1	o_2	o_3	o_4
f	5	2	10	2
g	0.8	0.7	1	0.7
h	27	1	100	1

There are infinitely many. Any order-preserving relabeling works.

This is what "**ordinal**" means: *only the order carries information*.

Consequence: utility cannot be compared across people.

Alice's utility for Massey's Columbus-style pizza is 5. Bob's is 4.

- Does Alice like Massey's more than Bob does? **No idea.**
- Bob could have written 400 instead of 4 and said the same thing
- Each person's numbers only speak about *that person's own* ranking

What Utility Does *Not* Mean

"Alice's utility for Cazuela's is 10."

- Meaningless on its own.

"Alice's utility for Cazuela's is 10 and for Yoshi Sushi is 5."

- Meaningful: it says she prefers Cazuela's.
- Does **not** say Cazuela's is "twice as good."
- We could have used 100 and -25 and said exactly the same thing.

Also: a utility of 1 does not mean "first choice." Low number, low rank.

Think of it as an index of satisfaction, but hold that thought loosely.

Careful: Two Things That Look Alike

Ordinal utility (now)	**Cardinal utility (Ch. 5)**
Only order matters	Differences matter too
Any increasing relabeling is an equally good U	Only positive affine transformations $aU + b$
Enough for dominance, Nash equilibrium	Needed for randomization and expected utility

For the next several weeks, we live entirely in the left column.

Definition: Ordinal Game in Strategic Form

An **ordinal game in strategic form** is

$$\langle I,\ (S_1, \ldots, S_n),\ O,\ f,\ (\succsim_1, \ldots, \succsim_n)\rangle$$

where

- $\langle I, (S_1, \ldots, S_n), O, f\rangle$ is a game-frame, and
- each $\succsim_i$ is a complete and transitive ranking of O

The only new ingredient is the list of rankings, one per player.

Payoff Functions

Replace each ranking $\succsim_i$ with a utility function U_i that represents it.

Then define player i's **payoff function** $\pi_i : S \to \mathbb{R}$ by

$$\pi_i(s) \;=\; U_i\big(f(s)\big)$$

- $f(s)$: the outcome that profile s produces
- $U_i(\cdot)$: how much i likes that outcome
- $\pi_i(s)$: the two composed (utility *as a function of strategies*)

The triple $\langle I, (S_1, \ldots, S_n), (\pi_1, \ldots, \pi_n)\rangle$ is the **reduced** ordinal game.
"Reduced" because we've thrown away the names of the outcomes.

The Pipeline

$$\begin{tikzcd} \boxed{\text{strategy profile } s} \arrow[r, "f"] \arrow[rr, dashed, bend right, "\pi_i"'] & \boxed{\text{outcome } f(s)} \arrow[r, "U_i"] & \boxed{\text{number } U_i(f(s))} \end{tikzcd}$$

Everything from here on is written in terms of π_i.
But remember what's underneath it.

Sunday Morning: Three Different Games, One Frame

Story A. Both would rather be together; Alex wants donuts, Blair wants chicken & waffles.

- Alex: $o_1 \succ_1 o_4 \succ_1 o_2 \sim_1 o_3$
- Blair: $o_4 \succ_2 o_1 \succ_2 o_2 \sim_2 o_3$

		Blair	
		BuckyDs	HangOE
Alex	BuckyDs	3, 2	1, 1
	HangOE	1, 1	2, 3

Sunday Morning: Three Different Games, One Frame

Story B. Alex wants company; Blair wants to eat alone this morning.

- Alex: $o_1 \sim_1 o_4 \succ_1 o_2 \sim_1 o_3$
- Blair: $o_2 \sim_2 o_3 \succ_2 o_1 \sim_2 o_4$

		Blair: BuckyDs	Blair: HangOE
Alex	BuckyDs	2, 0	0, 2
Alex	HangOE	0, 2	2, 0

Same frame. Completely different game.

Sunday Morning: Three Different Games, One Frame

Story C. Neither cares about company at all; each just wants their favorite food.

- Alex: $o_1 \sim_1 o_2 \succ_1 o_3 \sim_1 o_4$ (donuts, period)
- Blair: $o_2 \sim_2 o_4 \succ_2 o_1 \sim_2 o_3$ (chicken & waffles, period)

		Blair: BuckyDs	Blair: HangOE
Alex	BuckyDs	1, 0	1, 1
Alex	HangOE	0, 0	0, 1

Here the answer feels obvious. Why?

Back to Golden Balls

Suppose **both** are selfish and greedy:

- Sarah: $o_3 \succ_1 o_1 \succ_1 o_2 \sim_1 o_4$
- Steven: $o_2 \succ_2 o_1 \succ_2 o_3 \sim_2 o_4$

Pick utilities (any numbers with the right order will do):

	o_1	o_2	o_3	o_4
U_1	3	2	4	2
U_2	3	4	2	2

Sarah \ Steven	Split	Steal
Split	3, 3	2, 4
Steal	4, 2	2, 2

Back to Golden Balls

Now suppose Sarah is **fair-minded and benevolent,** Steven still selfish and greedy:

- Sarah: $o_1 \succ_1 o_3 \succ_1 o_2 \succ_1 o_4$
- Steven: $o_2 \succ_2 o_1 \succ_2 o_3 \sim_2 o_4$

	o_1	o_2	o_3	o_4
U_1	4	2	3	1
U_2	3	4	2	2

Sarah \ Steven	Split	Steal
Split	4, 3	2, 4
Steal	3, 2	1, 2

Same show, same rules, same table of prizes. **Different game.**

Dominance

The Idea

Fix one player. Compare two of *their own* strategies, a and b.

Ask: how do a and b compare **in every situation they might face**?

- If a always beats b: a **strictly dominates** b
- If a never loses to b,
  and sometimes beats it: a **weakly dominates** b
- If they always tie: a and b are **equivalent**

"Situation" $=$ a choice of strategies by *everyone else*.

Notation for "Everyone Else"

With n players, a profile is $s = (s_1, \ldots, s_n)$.

Fix a player i. Write

$$s_{-i} = \text{the strategies of all players other than } i$$

so that $s = (s_i,\ s_{-i})$.

- S_{-i} is the set of all such sub-profiles: $S_{-i} = \times_{j \neq i} S_j$
- $\pi_i(a, s_{-i})$ means: i plays a, everyone else plays s_{-i}

Example: $S_1 = \{a, b, c\}$, $S_2 = \{d, e\}$, $S_3 = \{f, g\}$, and $s = (b, d, g)$.

Then $s_{-2} =$ ____________________
and $(e, s_{-2}) =$ ____________________

An Example (Player 1's Payoffs Only)

		Player 2		
		L	C	R
Player 1	T	4, ·	3, ·	5, ·
	M	2, ·	1, ·	4, ·
	B	4, ·	1, ·	5, ·

Compare *T* and *M*, row by row:

- vs. *L*: 4 _____ 2
- vs. *C*: 3 _____ 1
- vs. *R*: 5 _____ 4

Conclusion:

An Example (Player 1's Payoffs Only)

		Player 2		
		L	C	R
Player 1	T	4, ·	3, ·	5, ·
	M	2, ·	1, ·	4, ·
	B	4, ·	1, ·	5, ·

Now compare *T* and *B*:

- vs. *L*: 4 _____ 4
- vs. *C*: 3 _____ 1
- vs. *R*: 5 _____ 5

T never does worse, and does better against *C*. This is **weak** dominance.

Definition: Dominance

Let i be a player and $a, b \in S_i$ two of their strategies.

- a **strictly dominates** b if

$$\pi_i(a, s_{-i}) > \pi_i(b, s_{-i}) \quad \text{for every } s_{-i} \in S_{-i}$$

- a **weakly dominates** b if

$$\pi_i(a, s_{-i}) \geq \pi_i(b, s_{-i}) \quad \text{for every } s_{-i} \in S_{-i}$$

$$\textit{and} \;\; \pi_i(a, s_{-i}) > \pi_i(b, s_{-i}) \text{ for at least one } s_{-i}$$

- a is **equivalent** to b if

$$\pi_i(a, s_{-i}) = \pi_i(b, s_{-i}) \quad \text{for every } s_{-i} \in S_{-i}$$

Two Conventions

1. Strict dominance technically satisfies the definition of weak dominance.

From here on, "*a* weakly dominates *b*" means *weakly but not strictly*.

2. "Dominated" is a comparison, so name the other strategy.

- "*x* is dominated" $\rightsquigarrow$ dominated by *what*?
- "*x* is dominated by *y*" $\rightsquigarrow$ fine

Think of "dominates" as "is better than."

Dominant $=$ Best

"Dominant" needs no second strategy named: it means *better than all of them*.

- a is a **strictly dominant strategy** if a strictly dominates *every* other strategy of player i
- a is a **weakly dominant strategy** if, for every other strategy x, either a weakly dominates x or a is equivalent to x

Consequences:

- There is at most **one** strictly dominant strategy. Why?
- There can be **several** weakly dominant strategies, but any two of them must be equivalent.

Dominant-Strategy Profiles

Let $s = (s_1, \ldots, s_n)$ be a strategy profile.

- s is a **strict dominant-strategy profile** if s_i is strictly dominant for *every* player i
- s is a **weak dominant-strategy profile** if s_i is weakly dominant for every player i, and for at least one player j, s_j is not strictly dominant

Unqualified “dominant-strategy profile” defaults to **weak**.

(You’ll also see “dominant-strategy equilibrium” in the literature.)

Your Turn

Only Player 1's payoffs are shown.

			Player 2	
		E	F	G
Player 1	A	3, ·	2, ·	1, ·
	B	2, ·	1, ·	0, ·
	C	3, ·	2, ·	1, ·
	D	2, ·	0, ·	0, ·

Classify each pair. Then: any weakly dominant strategies?

A vs. *B*:

A vs. *D*:

A vs. *C*:

B vs. *D*:

So What About Sarah?

Selfish and greedy version:

		Steven	
		Split	Steal
Sarah	Split	3, 3	2, 4
	Steal	4, 2	2, 2

- Steal is a ________________ dominant strategy for each player
- So (Steal, Steal) is a ________________ dominant-strategy profile

And the fair-minded version?

What Did *We* Do?

Back to our MobLab results.

Chose Split: ________________

Chose Steal: ________________

If everyone were selfish and greedy, Steal is weakly dominant, so we'd expect 100% Steal.

We didn't get that. So which assumption was wrong?

Let’s Watch

Golden Balls: Split or Steal

https://www.youtube.com/watch?v=tBtr8-VMj0E

While you watch, ask yourself:

- Which game are *they* playing?
- What does the talking do, if agreements aren’t binding?

Our Own Example: The Price War

Dos Hermanos and Pitabilities park on the same block every day. Each morning, each owner independently sets a **Normal** price or runs a **Discount**.

- If neither discounts: they split the lunch crowd at healthy margins
- If exactly one discounts: that truck pulls most of the crowd
- If both discount: they split the same crowd at thin margins

Each owner ranks the four outcomes:

$$\text{(I discount, they don't)} \succ \text{(neither)} \succ \text{(both)} \succ \text{(they discount, I don't)}$$

The Price War

		Pitabilities	
		Normal	Discount
Dos Hermanos	Normal	$2, 2$	$0, \underline{3}$
	Discount	$\underline{3}, 0$	$\underline{1}, \underline{1}$

- Discount ______________________ dominates Normal, for both
- So (Discount, Discount) is a ______________________ profile

But look at (Normal, Normal):

Definition: Pareto Superiority

Let o and o' be two outcomes.

- o is **strictly Pareto superior** to o' if *every* player prefers o to o': $o \succ_i o'$ for all i
- o is **weakly Pareto superior** to o' if every player finds o at least as good, and at least one strictly prefers it

In reduced games, same thing with payoffs: $\pi_i(s) > \pi_i(s')$ for all i, etc.

In the Price War, (Normal, Normal) is strictly Pareto superior to (Discount, Discount).

Yet rational play lands on (Discount, Discount).

The Prisoner's Dilemma

That structure has a name. Bonanno's version:

Doug and Ed are the only two candidates for a best-worker prize, currently tied. Unless one outperforms the other, nobody gets it. Each chooses Normal or Extra effort.

- o_1: nobody gets the prize, nobody sacrifices family time
- o_2: Ed gets the prize and sacrifices family time, Doug does not
- o_3: Doug gets the prize and sacrifices family time, Ed does not
- o_4: nobody gets the prize, both sacrifice family time

Both are willing to sacrifice family time for the prize, otherwise value it, and are envious: $o_3 \succ_{\text{Doug}} o_1 \succ_{\text{Doug}} o_4 \succ_{\text{Doug}} o_2$.

The Prisoner's Dilemma

		Ed	
		Normal	Extra
Doug	Normal	2, 2	0, $\underline{3}$
	Extra	$\underline{3}$, 0	$\underline{1}$, $\underline{1}$

Extra effort is strictly dominant for both.
(Extra, Extra) is a strict dominant-strategy profile.

Same game as the price war: different story, identical structure.
That is what game theory is for.

Individual vs. Collective Rationality

A strictly dominant strategy is not a suggestion. Playing anything else means a lower payoff *no matter what the other player does*.

So rational individual play $\Rightarrow$ (Extra, Extra), even though both prefer (Normal, Normal).

Couldn't they just agree?

- Non-cooperative game theory assumes agreements are **not binding**
- If I expect you to keep it, I gain by breaking it
- If I expect you to break it, I gain by breaking it too

Let's Work One Out

Three Carts at the North Market

Ann, Ben, and Cal each run a cart. Every Saturday each one independently picks a special: the **Classic** or something **New**.

Cal: Classic

Ann \ Ben	Classic	New
Classic	3, 3, 2	4, 3, 1
New	1, 3, 4	2, 1, 3

Cal: New

Ann \ Ben	Classic	New
Classic	3, 2, 3	4, 0, 2
New	1, 2, 3	2, 0, 4

Ann picks the row, Ben the column, Cal the table. Payoffs in order: Ann, Ben, Cal.

Three Carts at the North Market

1. Does Ann have a dominant strategy? Strict or weak?
2. Does Ben?
3. Does Cal?
4. Is there a dominant-strategy profile?
5. If Ann and Ben play their dominant strategies, what should Cal do?

What Have We Learned, Charlie Brown?

- A **game-frame** is players, strategies, outcomes, and f
- Preferences are a complete, transitive relation $\succsim_i$ on outcomes
- An **ordinal utility function** represents $\succsim_i$; the numbers are arbitrary
- Frame $+$ preferences $=$ **game**; composing gives payoffs $\pi_i = U_i \circ f$
- **Dominance** compares two strategies of one player across all situations
- A dominant strategy is best against everything

Next: what if nobody has a dominant strategy?

Reading & Practice

- Bonanno, *Game Theory*, §2.1–2.2
- Exercises: §2.9.1 (frames and preferences), §2.9.2 (dominance)
- Full solutions are in §2.10. Try them before you look