

Math Review (Not in the book)

ECON 5001

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Why a Math Review?

- Official prereqs: ECON 4001.0x and MATH 1151 (Calc 1)
- Reality: huge range of experience in the room
- Also, some non-calculus concepts
- Nothing too hard! (IMO)

A **set** is simply a collection of things.

- Example: $A = \{\text{rock}, \text{paper}, \text{scissors}\}$
 - Shorthand names: $A = \{r, p, c\}$
- Use curly brackets (“braces”)
- Comma-separated list
- Order doesn’t matter
- No repetition (this isn’t so important. Repetition $\Rightarrow$ “multiset”)

- “Singleton” set:
- Empty set:

Set Operations

- Subset: $A \subseteq B$ and $A \subset B$
- Union: $A \cup B$
- Intersection: $A \cap B$
- Set difference (“set minus”): $A \setminus B$

Set Operations

- Universal set: set containing “all things” (e.g., possible die rolls) (only things we’re currently talking about)
- Complement: $A^c = U \setminus A$
- Element of: $\in$ (and not an element of: $\notin$)
- “Disjoint” (mutually exclusive) sets: $A \cap B = \emptyset$

Math Language: Quantifiers

- “For all” (or “for every”): $\forall$
- “There exists ___ such that ___”: $\exists$ ___ s.t. ___

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

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Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \exists y \in B \text{ s.t. } y > x$
When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ ✓

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Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \exists y \in B \text{ s.t. } y > x$
When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ ✓
- Is this true: $\exists x \in B \text{ s.t. } \forall y \in A \quad x > y$
Set $x = 4$. When $y = 1$ then $x > y$, and when $y = 3$ then $x > y$ ✓

Math Language: Quantifiers

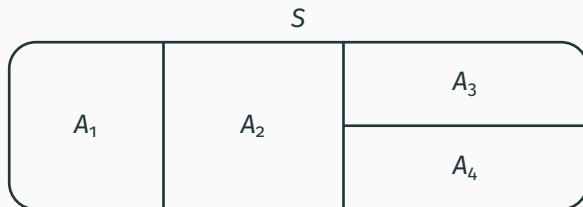
- “For all” (or “for every”): $\forall$
- “There exists ___ such that ___”: $\exists$ ___ s.t. ___

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \exists y \in B$ s.t. $y > x$
When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ ✓
- Is this true: $\exists x \in B$ s.t. $\forall y \in A$ $x > y$
Set $x = 4$. When $y = 1$ then $x > y$, and when $y = 3$ then $x > y$ ✓
- Is this true: $\forall x \in B \exists y \in A$ s.t. $y > x$
What do you think?

Partitions

A **partition** of a set S carves it up with no gaps and no overlaps.



Each A_j is a **cell**. The whole collection $\{A_1, A_2, A_3, A_4\}$ is the **partition**. (A set of sets!)

Partitions

Three requirements. In words, then in symbols:

- every cell is **nonempty**: $A_j \neq \emptyset$
- cells are **mutually exclusive**, no two overlap:

$$\underbrace{A_j \cap A_k = \emptyset}_{\text{no element is in two cells}} \quad \text{for all } j \neq k$$

- cells **cover** S , together they use up everything:

$$\underbrace{A_1 \cup A_2 \cup \dots \cup A_m}_{\text{everything in some cell}} = S$$

So every element of S is in **exactly one** cell.

Implies, and If and Only If

$P \Rightarrow Q$ reads “ P implies Q ” or “if P , then Q ”

- whenever P is true, Q must be true
- it says **nothing** about what happens when P is false

$P \Leftrightarrow Q$ reads “ P if and only if Q ”

- this is $P \Rightarrow Q$ and $Q \Rightarrow P$, both directions
- P and Q are true in exactly the same situations

Implies, and If and Only If

$$x = 2 \implies x^2 = 4$$

True. But the arrow does not reverse:

$$x^2 = 4 \iff (x = 2 \text{ or } x = -2)$$

True in both directions.

Cartesian Product

The set $S = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$ contains **all possible combinations** of $A = \{r, p, c\}$ and $B = \{r, p, c\}$. This is called the **Cartesian product** (or just **product**) of A and B .

Notation: $A \times B$.

Set B	r			
	p			
	c			
		r	p	c
		Set A		

Cartesian Product

The set $S = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$ contains **all possible combinations** of $A = \{r, p, c\}$ and $B = \{r, p, c\}$. This is called the **Cartesian product** (or just **product**) of A and B .

Notation: $A \times B$.

Set B	r	(r, r)	(p, r)	(c, r)
	p	(r, p)	(p, p)	(c, p)
	c	(r, c)	(p, c)	(c, c)
		r	p	c
		Set A		

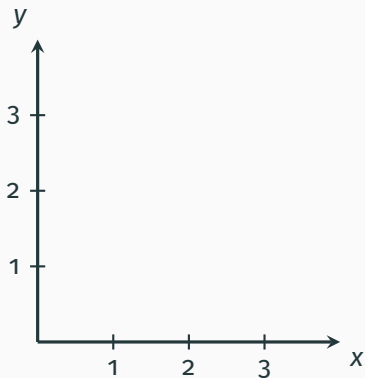
Cartesian Product

You've seen Cartesian products before:

Set B	r	(r, r)	(p, r)	(c, r)
	p	(r, p)	(p, p)	(c, p)
	c	(r, c)	(p, c)	(c, c)
		r	p	c
		Set A		

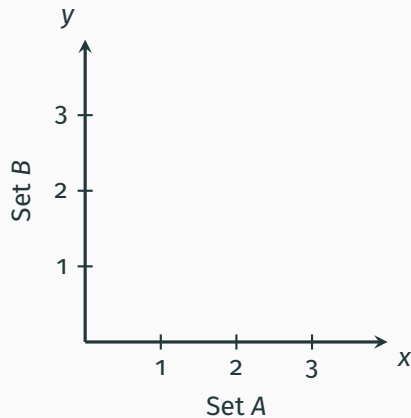
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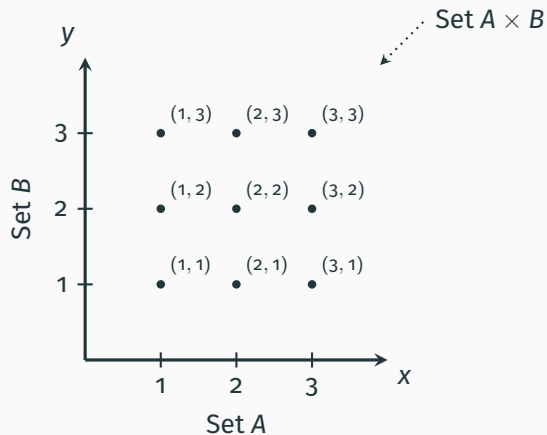
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Cartesian Product

You've seen Cartesian products before:



Indexed Sets

Sets may be numbered (“indexed”). Example:

Player 1's strategies: $S_1 = \{a, b\}$

Player 2's strategies: $S_2 = \{c, d\}$

Player 3's strategies: $S_3 = \{x, y\}$

We can define $S = S_1 \times S_2 \times S_3$

We can do this for any number n sets: $S = S_1 \times S_2 \times \cdots \times S_n$

Shorthand notation: $S = \times_{i=1}^n S_i$

Lists

Suppose $S_1 = \{r, p, c\}$ and $S_2 = \{r, p, c\}$.

So $S_1 \times S_2 = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$.

Elements of this set are **lists**.

Other names: **ordered pairs** (if 2), ***n*-tuples** (if *n*), **vectors**, **profiles**

List (r, c) means P1 played Rock and P2 played Scissors. Order matters!!

Sets vs. Lists:

Sets	Lists
Ex: $\{a, b, c, d\}$	Ex: (a, d, d, a)
Braces $\{\dots\}$	Parentheses $(\dots)$
Comma-separated	Comma-separated
Order doesn't matter	Order matters!!!
Cannot repeat	Can repeat

Confusing: $(0, 10)$ can represent a list or an interval. Context should clarify.

Cartesian Products & Lists

Rock-Paper-Scissors:

$$S_1 = \{r, p, c\}$$

$$S_2 = \{r, p, c\}$$

$S = S_1 \times S_2$ (Cartesian product)

$$S = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$$

Each element of S is a **list**

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List says (1) what P1 chose in S_1 , (2) what P2 chose in S_2

In game theory, lists of strategies are **strategy profiles**. e.g. (r, c) .

Strategy Profiles With n Players

Game theory notation:

- Suppose a game has n players

- What does this mean: "Each player i chooses some $s_i \in S_i$ "

- Any particular player number: $i \in \{1, 2, \dots, n\}$

- Set of possible strategies for player i : $S_i \leftarrow i$'s strategy set eg. $\{r, l, c\}$

- A particular strategy for player i : $s_i \leftarrow i$'s chosen strategy eg. r

Player 1 chooses $s_1 \in S_1$
Player 2 chooses $s_2 \in S_2$
:
Player n chooses $s_n \in S_n$

Strategy profile $s = (s_1, s_2, \dots, s_n) \in S = S_1 \times S_2 \times \dots \times S_n$

$\uparrow$ $\uparrow$
 p_1 p_2

Strategy profile

vs,

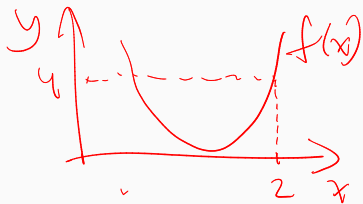
strategy

“Not i ” Notation: s_{-i}

- Suppose there are n players
- Each player i plays strategy s_i
- The profile is $s = (s_1, s_2, s_3, \dots, s_n)$ ← strategy profile
- Suppose we want to focus on player 7
 - “I am player 7”, so set $i = 7$
- In this case, $i = 7$. So $s_i = s_7$
- The strategies of others are $s_{-7} = (s_1, s_2, \dots, s_6, s_8, \dots, s_n)$ ← s_7 removed
↓
profile of all others' strategies
- For any player i , call this s_{-i}
 - $s_{-7} = (s_1, s_2, \dots, s_6, s_8, \dots, s_n)$
 - $s_{-3} = (s_1, s_2, s_4, \dots, s_n)$
 - $s_{-1} = (s_2, s_3, \dots, s_n)$
- The entire profile can be written as $s = (s_i, s_{-i})$ for any arbitrary i
 - s_i is what player i plays
 - s_{-i} is what all the other players play

Functions

Function you're probably used to: $y = x^2$ or $f(x) = x^2$



What is this function doing? Mapping one set into another!

$$\text{Domain} \left\{ \begin{array}{l} 2 \rightarrow 4 \\ 3 \rightarrow 9 \\ -2 \rightarrow 4 \end{array} \right\} \text{Range}$$

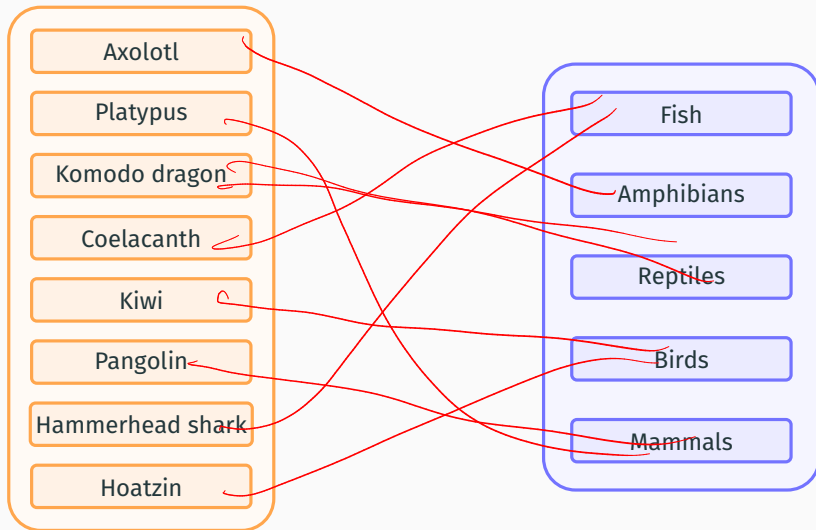
Domain vs. Range (or “Codomain”)

More General Functions

Any domain and any range. Ex: classes of animals.

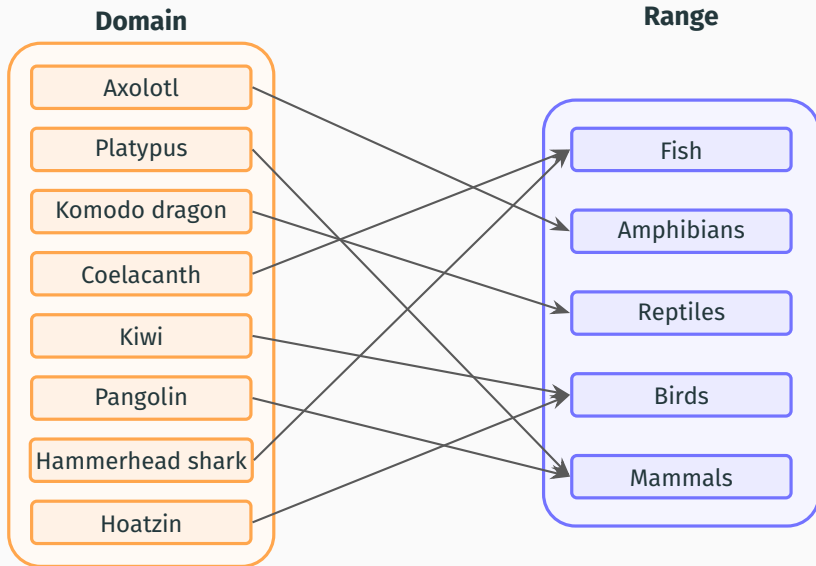
Domain

Range



More General Functions

Any domain and any range. Ex: classes of animals.



A Game Theory Example

Rock-Paper-Scissors:

9 possible strategy pairs: $S = \left\{ \begin{array}{ccc} (r,r), & (r,p), & (r,c), \\ (p,r), & (p,p), & (p,c), \\ (c,r), & (c,p), & (c,c) \end{array} \right\}$

First letter: P1's choice. Second letter: P2's choice

Outcomes: $O = \{P1Wins, Tie, P2Wins\}$

Domain
Inputs

Outcome
function

Domain $\left\{ \begin{array}{l} (r,r) \rightarrow Tie \\ (r,p) \rightarrow P2 \\ (r,c) \rightarrow P1 \\ (p,r) \rightarrow P1 \\ \vdots \end{array} \right\}$ Range

"Outcome function"

A Game Theory Example

We can represent this in table format.

Convention: Player 1 is rows, Player 2 is columns

Strategy ~~pairs~~ ^{profiles}:

		Set S_2		
		r	p	c
Set S_1	r	(r, r)	(r, p)	(r, c)
	p	(p, r)	(p, p)	(p, c)
	c	(c, r)	(c, p)	(c, c)

→

Outcome function:

		r	p	c
	r	T	P_2	
	p		T	
	c			T

P_1 chooses row
 P_2 choose column

A Game Theory Example

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Strategy ~~pairs~~ ^{profile}:

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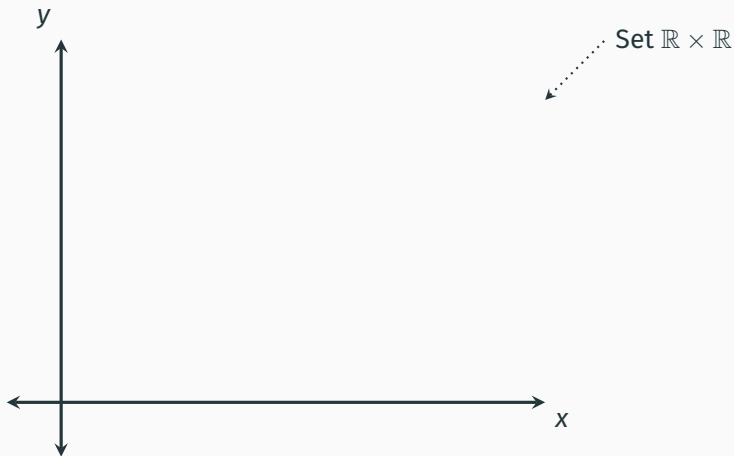
Outcome function:

	r	p	c
r	Tie	P2	P1
p	P1	Tie	P2
c	P2	P1	Tie

A Bit of Calculus

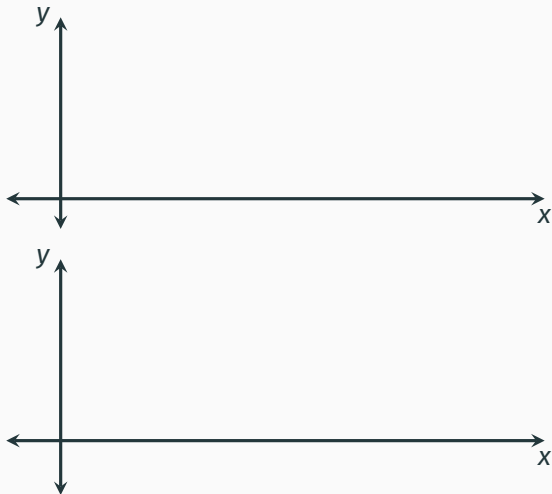
Set of all “real” numbers: $\mathbb{R} = (-\infty, \infty)$ (interval)

“Real” function: domain and range are both $\mathbb{R}$



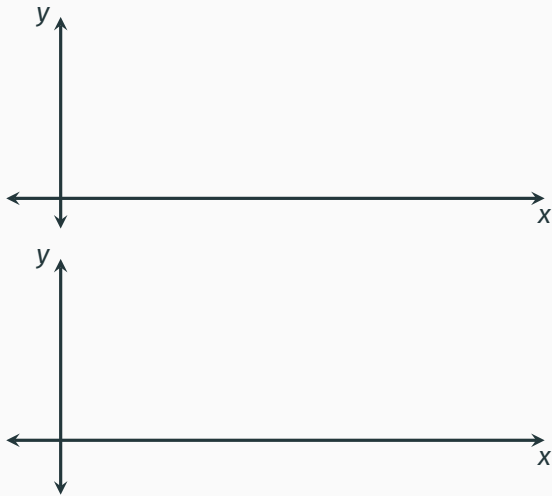
A Bit of Calculus

What is the slope at a point?



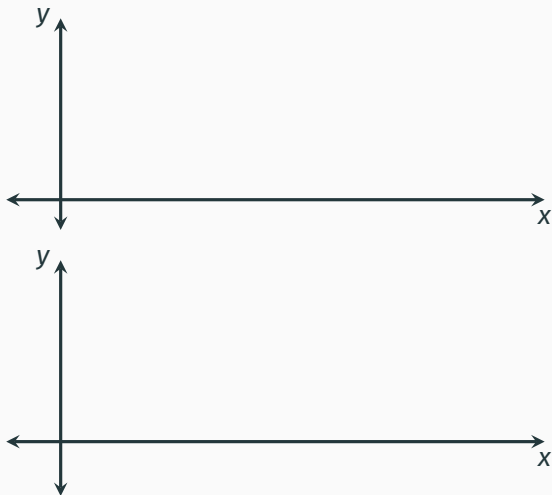
A Bit of Calculus

What is the derivative?



A Bit of Calculus

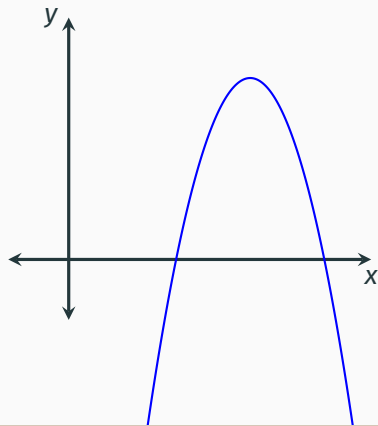
How to find the max of a function using calculus?



A Bit of Calculus

A worked example.

- Function: $f(x) = -4 + 12x - 2x^2$
- Derivative: $f'(x) =$
- Set equal to zero:
- Solve:



That's enough for now!

Please ask questions if ever some math notation is unclear.