

Math Review (Not in the book)

ECON 5001

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Why a Math Review?

- Official prereqs: ECON 4001.0x and MATH 1151 (Calc 1)
- Reality: huge range of experience in the room
- Also, some non-calculus concepts
- Nothing too hard! (IMO)

A **set** is simply a collection of things.

- Example: $A = \{\text{rock}, \text{paper}, \text{scissors}\}$ *← strategies*
 - Shorthand names: $A = \{r, p, c\}$
- Use curly brackets (“braces”) *{ }*
- Comma-separated list
- Order doesn't matter
- No repetition (this isn't so important. Repetition $\Rightarrow$ “multiset”)

- “Singleton” set: *$\{r\}$ one object*

- Empty set:

$$\emptyset = \{ \} \quad \text{nothing}$$

Set Operations

- Subset: $A \subseteq B$ and $A \subset B$

could be same
not same

$$\{r\} \subseteq \{r, p, c\}$$

$$\{r\} \subseteq \{r\}$$

$$\{r\} \subset \{r, p, c\}$$

- Union: $A \cup B$

$$\{r\} \cup \{p, c\} = \{r, p, c\}$$

- Intersection: $A \cap B$

$$\{r, p\} \cap \{p, c\} = \{p\}$$

- Set difference ("set minus"): $A \setminus B$

$$\{r, p, c\} \setminus \{c\} = \{r, p\}$$

Set Operations

- Universal set: set containing “all things” (e.g., possible die rolls)
(only things we’re currently talking about)

$$U = \{r, p, c\}$$

- Complement: $A^c = U \setminus A$

$$A = \{r, p\} \quad A^c = \{r, p, c\} \setminus \{r, p\} \Rightarrow A^c = \{c\}$$

- Element of: $\in$ (and not an element of: $\notin$)

$$r \in \{r, p\}$$

$$A = \{r, p\}$$

$$r \in A$$

- “Disjoint” (mutually exclusive) sets: $A \cap B = \emptyset$

$$\{r\} \cap \{p, c\} = \emptyset$$

disjoint

Math Language: Quantifiers

- “For all” (or “for every”): $\forall$
- “There exists ___ such that ___”: $\exists$ ___ s.t. ___

$\forall$

$\exists$

s.t.

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

Math Language: Quantifiers

- “For all” (or “for every”): $\forall$
- “There exists ___ such that ___”: $\exists$ ___ s.t. ___

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \exists y \in B$ s.t. $y > x$ **TRUE**
When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ ✓

$x = 1$	$y = 4$ ✓	✓
$x = 3$	$y = 4$ ✓	

Math Language: Quantifiers

- “For all” (or “for every”): $\forall$
- “There exists ___ such that ___”: $\exists$ ___ s.t. ___

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \exists y \in B$ s.t. $y > x$
When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ ✓
- Is this true: $\exists x \in B$ s.t. $\forall y \in A$ $x > y$ **TRUE**
Set $x = 4$. When $y = 1$ then $x > y$, and when $y = 3$ then $x > y$ ✓

$x=2$? No

$y=1$ ✓

$y=3$ ✗ $y > x$

$x=4$?

$y=1$ ✓ $4 > 1$

$y=3$ ✓ $4 > 3$

yes

Math Language: Quantifiers

- “For all” (or “for every”): $\forall$
- “There exists ___ such that ___”: $\exists$ ___ s.t. ___

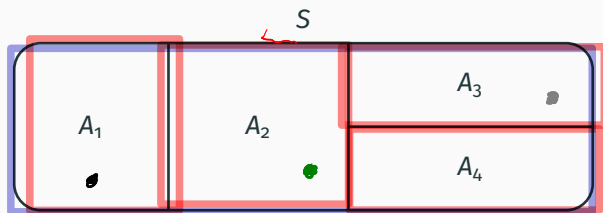
Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \exists y \in B$ s.t. $y > x$
When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ ✓
- Is this true: $\exists x \in B$ s.t. $\forall y \in A$ $x > y$
Set $x = 4$. When $y = 1$ then $x > y$, and when $y = 3$ then $x > y$ ✓
- Is this true: $\forall x \in B \exists y \in A$ s.t. $y > x$ **NO**
What do you think?

$x = -1$ $y = 1$ ✓
 $x = 2$ $y = 3$ ✓
 $x = 4$ $y = 1$ ✗
 $y = 3$ ✗ NO

Partitions

A **partition** of a set S carves it up with no gaps and no overlaps.

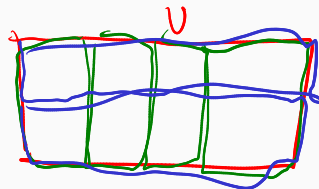


Not
partition



Each A_j is a **cell**. The whole collection $\{A_1, A_2, A_3, A_4\}$ is the **partition**. (A set of sets!)

partition
subsets



Partitions

Three requirements. In words, then in symbols:

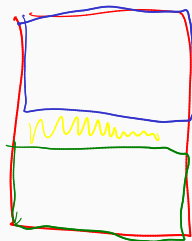
- every cell is **nonempty**: $A_j \neq \emptyset$ ✓
- cells are **mutually exclusive**, no two overlap:

$$\underbrace{A_j \cap A_k = \emptyset}_{\text{no element is in two cells}} \quad \text{for all } j \neq k \quad \checkmark$$

- cells **cover** S , together they use up everything:

$$\underbrace{A_1 \cup A_2 \cup \dots \cup A_m}_{\text{everything in some cell}} = S$$

So every element of S is in **exactly one** cell.



Implies, and If and Only If

$P \Rightarrow Q$ reads “ P implies Q ” or “if P , then Q ”

- whenever P is true, Q must be true
- it says **nothing** about what happens when P is false



$P \Leftrightarrow Q$ reads “ P if and only if Q ”

- this is $P \Rightarrow Q$ and $Q \Rightarrow P$, both directions
- P and Q are true in exactly the same situations



Implies, and If and Only If

$$\begin{array}{c} P \\ \circlearrowleft \\ x = 2 \end{array} \implies \begin{array}{c} Q \\ \circlearrowleft \\ x^2 = 4 \end{array}$$

$$x=3 \not\Rightarrow x^2=4$$

$$x=-2 \Rightarrow x^2=4$$

True. But the arrow does not reverse:

$$\begin{array}{c} Q \\ \circlearrowleft \\ x^2 = 4 \end{array} \iff \begin{array}{c} P \\ \circlearrowleft \\ (x = 2 \text{ or } x = -2) \end{array}$$

True in both directions.

Cartesian Product

The set $S = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$ contains **all possible combinations** of $A = \{r, p, c\}$ and $B = \{r, p, c\}$. This is called the **Cartesian product** (or just **product**) of A and B .

Notation: $A \times B$.

Player 2

Set B	r	r, r	r, p	r, c
	p	p, r		
	c			
		r	p	c

Set A

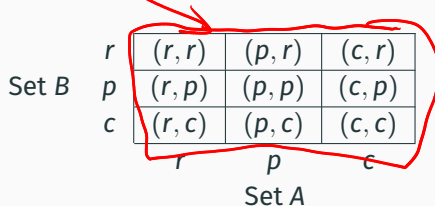
Player 1

Cartesian Product

The set $S = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$ contains **all possible combinations** of $A = \{r, p, c\}$ and $B = \{r, p, c\}$

This is called the **Cartesian product** (or just **product**) of A and B

Notation: $A \times B$.



Set B	r	(r, r)	(p, r)	(c, r)
	p	(r, p)	(p, p)	(c, p)
	c	(r, c)	(p, c)	(c, c)
		r	p	c
		Set A		

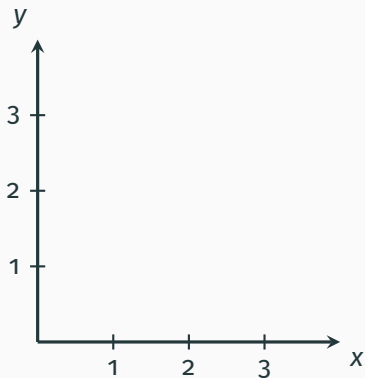
Cartesian Product

You've seen Cartesian products before:

Set B	r	(r, r)	(p, r)	(c, r)
	p	(r, p)	(p, p)	(c, p)
	c	(r, c)	(p, c)	(c, c)
		r	p	c
		Set A		

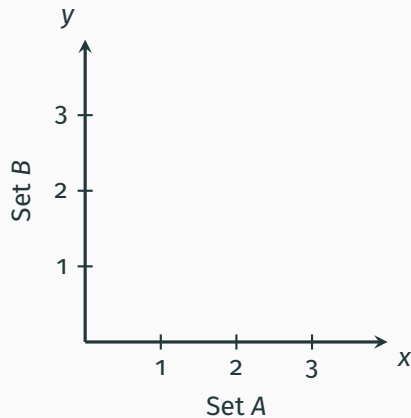
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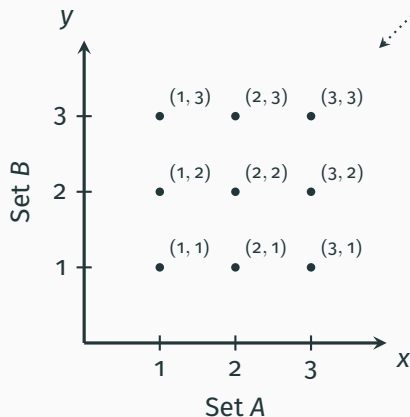
Cartesian Product

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Cartesian Product

You've seen Cartesian products before:



Set $A \times B$

A "cross" B

*Cartesian product
of A and B*

Indexed Sets

Sets may be numbered ("indexed"). Example:

Player 1's strategies: $S_1 = \{a, b\}$

Player 2's strategies: $S_2 = \{c, d\}$

Player 3's strategies: $S_3 = \{x, y\}$

We can define $S = S_1 \times S_2 \times S_3$

$\longrightarrow S_1$

$$S = \{(a, c, x), (a, c, y), (a, d, x), (a, d, y), \dots\}$$

We can do this for any number n sets: $S = S_1 \times S_2 \times \dots \times S_n$

Shorthand notation: $S = \times_{i=1}^n S_i$

Lists

Suppose $S_1 = \{r, p, c\}$ and $S_2 = \{r, p, c\}$.

So $S_1 \times S_2 = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$.

Elements of this set are **lists**.

Other names: **ordered pairs** (if 2), ***n*-tuples** (if *n*), **vectors**, **profiles**

List (r, c) means P1 played Rock and P2 played Scissors. Order matters!!

(c, r)

Game Theory
language

Sets vs. Lists:

Sets	Lists
Ex: $\{a, b, c, d\}$	Ex: (a, d, d, a)
Braces $\{\dots\}$	Parentheses $(\dots)$
Comma-separated	Comma-separated
Order doesn't matter	Order matters!!!
Cannot repeat	Can repeat

Confusing: $(0, 10)$ can represent a list or an interval. Context should clarify.

Cartesian Products & Lists

Rock-Paper-Scissors:

$$S_1 = \{r, p, c\}$$

$$S_2 = \{r, p, c\}$$

$S = S_1 \times S_2$ (Cartesian product)

$$S = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$$

Each element of S is a **list**

List says (1) what P_1 chose in S_1 , (2) what P_2 chose in S_2

(r, c)
 $\uparrow \quad \uparrow$
 $p_1 \quad p_2$

In game theory, lists of strategies are **strategy profiles**. e.g. (r, c) .

Strategy Profiles With n Players

Game theory notation:

- Suppose a game has n players
- What does this mean: “Each player i chooses some $s_i \in S_i$ ”
- Any particular player number: $i \in \{1, 2, \dots, n\}$
- Set of possible strategies for player i : S_i
- A particular strategy for player i : s_i

Player 1 chooses $s_1 \in S_1$

Player 2 chooses $s_2 \in S_2$

$\vdots$

Player n chooses $s_n \in S_n$

Strategy profile $s = (s_1, s_2, \dots, s_n) \in S = S_1 \times S_2 \times \dots \times S_n$

“Not i ” Notation: s_{-i}

- Suppose there are n players
- Each player i plays strategy s_i
- The profile is $s = (s_1, s_2, s_3, \dots, s_n)$
- Suppose we want to focus on player 7
 - “I am player 7”, so set $i = 7$
- In this case, $i = 7$. So $s_i = s_7$
- The strategies of others are $s_{-7} = (s_1, s_2, \dots, s_6, s_8, \dots, s_n)$
- For any player i , call this s_{-i}
 - $s_{-7} = (s_1, s_2, \dots, s_6, s_8, \dots, s_n)$
 - $s_{-3} = (s_1, s_2, s_4, \dots, s_n)$
 - $s_{-1} = (s_2, s_3, \dots, s_n)$
- The entire profile can be written as $s = (s_i, s_{-i})$ for any arbitrary i
 - s_i is what player i plays
 - s_{-i} is what all the other players play

Functions

Function you're probably used to: $y = x^2$ or $f(x) = x^2$

What is this function doing? Mapping one set into another!

Domain vs. Range (or “Codomain”)

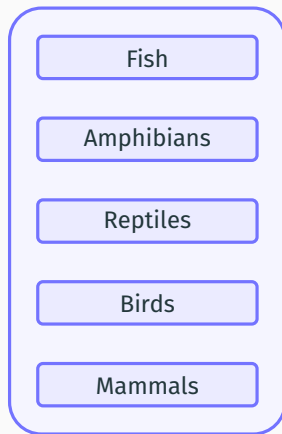
More General Functions

Any domain and any range. Ex: classes of animals.

Domain

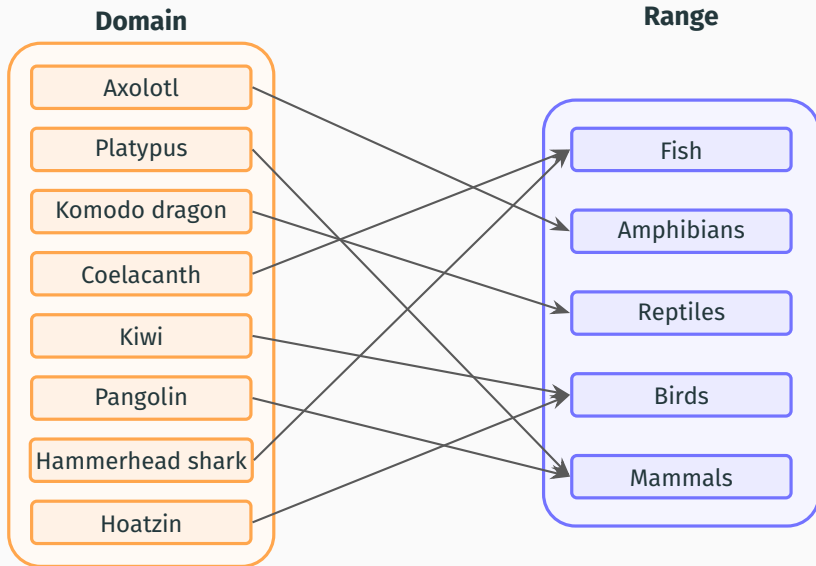


Range



More General Functions

Any domain and any range. Ex: classes of animals.



A Game Theory Example

Rock-Paper-Scissors:

9 possible strategy pairs: $S = \left\{ \begin{array}{ccc} (r,r), & (r,p), & (r,c), \\ (p,r), & (p,p), & (p,c), \\ (c,r), & (c,p), & (c,c) \end{array} \right\}$

First letter: P1's choice. Second letter: P2's choice

Outcomes: $O = \{P1Wins, Tie, P2Wins\}$

“Outcome function”

A Game Theory Example

We can represent this in table format.

Convention: Player 1 is rows, Player 2 is columns

Strategy pairs:

		Set S_2		
		r	p	c
Set S_1	r	(r, r)	(r, p)	(r, c)
	p	(p, r)	(p, p)	(p, c)
	c	(c, r)	(c, p)	(c, c)

→

Outcome function:

		r	p	c
	r			
	p			
	c			

A Game Theory Example

We can represent this in table format.

Convention: Player 1 is rows, Player 2 is columns

Strategy pairs:

		Set S_2		
		r	p	c
Set S_1	r	(r, r)	(r, p)	(r, c)
	p	(p, r)	(p, p)	(p, c)
	c	(c, r)	(c, p)	(c, c)

→

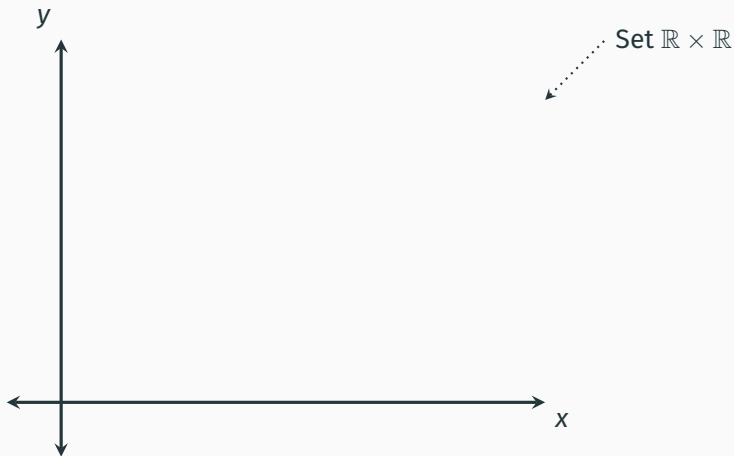
Outcome function:

		r	p	c
		r	p	c
	r	Tie	P2	P1
	p	P1	Tie	P2
	c	P2	P1	Tie

A Bit of Calculus

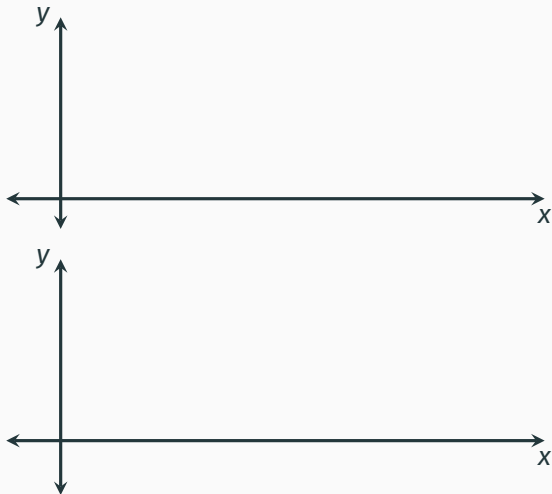
Set of all “real” numbers: $\mathbb{R} = (-\infty, \infty)$ (interval)

“Real” function: domain and range are both $\mathbb{R}$



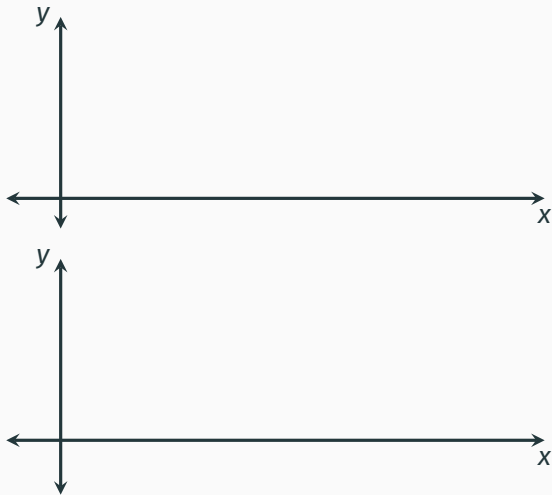
A Bit of Calculus

What is the slope at a point?



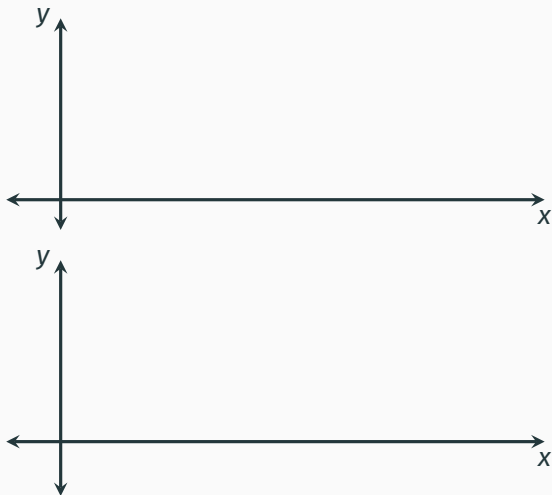
A Bit of Calculus

What is the derivative?



A Bit of Calculus

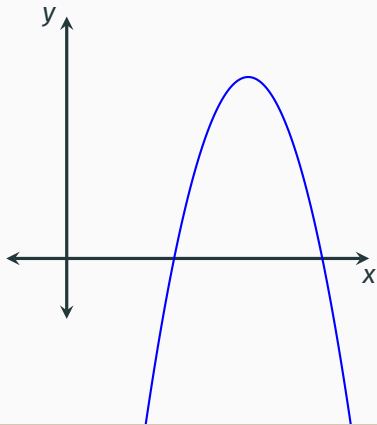
How to find the max of a function using calculus?



A Bit of Calculus

A worked example.

- Function: $f(x) = -4 + 12x - 2x^2$
- Derivative: $f'(x) =$
- Set equal to zero:
- Solve:



That's enough for now!

Please ask questions if ever some math notation is unclear.