Math Review (Not in the book)

ECON 5001
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Why a Math Review?

- Official prereqs: ECON 4001.0x and MATH 1151 (Calc 1)
- Reality: huge range of experience in the room
- Also, some non-calculus concepts
- Nothing too hard! (IMO)

Sets

A **set** is simply a collection of things.

- Example: $A = \{\text{rock,paper,scissors}\}$
    - Shorthand names: $A = \{r, p, c\}$
- Use curly brackets (“braces”)
- Comma-separated list
- Order doesn’t matter
- No repetition (this isn’t so important. Repetition $\Rightarrow$ “multiset”)

- “Singleton” set:
- Empty set:

Set Operations

- Subset: $A \subseteq B$ and $A \subset B$
- Union: $A \cup B$
- Intersection: $A \cap B$
- Set difference ("set minus"): $A \setminus B$

Set Operations

- Universal set: set containing "all things" (e.g., possible die rolls) (only things we're currently talking about)
- Compl**e**ment: $A^c = U \setminus A$
- Element of: $\in$ (and not an element of: $\notin$)
- "Disjoint" (mutually exclusive) sets: $A \cap B = \emptyset$

Math Language: Quantifiers

- "For all" (or "for every"): $\forall$
- "There exists __ such that __": $\exists$ __ s.t. __

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

Math Language: Quantifiers

- "For all" (or "for every"): $\forall$
- "There exists __ such that __": $\exists$ __ s.t. __

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \;\; \exists y \in B \;\; s.t. \quad y > x$
  When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ $\checkmark$

Math Language: Quantifiers

- "For all" (or "for every"): $\forall$
- "There exists __ such that __": $\exists$ __ s.t. __

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \;\; \exists y \in B \;\; s.t. \quad y > x$
  When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ ✓
- Is this true: $\exists x \in B \;\; s.t. \;\; \forall y \in A \quad x > y$
  Set $x = 4$. When $y = 1$ then $x > y$, and when $y = 3$ then $x > y$ ✓

Math Language: Quantifiers

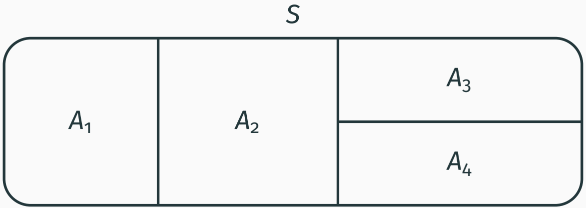
- "For all" (or "for every"): $\forall$
- "There exists __ such that __": $\exists$ __ s.t. __

Examples: Suppose $A = \{1, 3\}$, $B = \{-1, 2, 4\}$

- Is this true: $\forall x \in A \;\; \exists y \in B \;\; s.t. \quad y > x$
  When $x = 1$ set $y = 2$. When $x = 3$ set $y = 4$ ✓
- Is this true: $\exists x \in B \;\; s.t. \;\; \forall y \in A \quad x > y$
  Set $x = 4$. When $y = 1$ then $x > y$, and when $y = 3$ then $x > y$ ✓
- Is this true: $\forall x \in B \;\; \exists y \in A \;\; s.t. \quad y > x$
  What do you think?

Partitions

A **partition** of a set S carves it up with no gaps and no overlaps.



Each A_j is a **cell**. The whole collection $\{A_1, A_2, A_3, A_4\}$ is the **partition**. (A set of sets!)

Partitions

Three requirements. In words, then in symbols:

- every cell is **nonempty**: $A_j \neq \emptyset$
- cells are **mutually exclusive**, no two overlap:

$$\underbrace{A_j \cap A_k = \emptyset}_{\text{no element is in two cells}} \qquad \text{for all } j \neq k$$

- cells **cover** S, together they use up everything:

$$\underbrace{A_1 \cup A_2 \cup \cdots \cup A_m}_{\text{everything in some cell}} = S$$

So every element of S is in **exactly one** cell.

Implies, and If and Only If

$P \Rightarrow Q$ reads "P implies Q" or "if P, then Q"

- whenever P is true, Q must be true
- it says **nothing** about what happens when P is false

$P \Leftrightarrow Q$ reads "P if and only if Q"

- this is $P \Rightarrow Q$ *and* $Q \Rightarrow P$, both directions
- P and Q are true in exactly the same situations

Implies, and If and Only If

$$x = 2 \implies x^2 = 4$$

True. But the arrow does not reverse:

$$x^2 = 4 \iff (x = 2 \text{ or } x = -2)$$

True in both directions.

Cartesian Product

The set $S = \{(r,r),(r,p),(r,c),(p,r),(p,p),(p,c),(c,r),(c,p),(c,c)\}$ contains **all possible combinations** of $A = \{r,p,c\}$ and $B = \{r,p,c\}$

This is called the **Cartesian product** (or just **product**) of A and B

Notation: $A \times B$.

	r			
Set B	p			
	c			
		r	p	c
			Set A	

Cartesian Product

The set $S = \{(r,r),(r,p),(r,c),(p,r),(p,p),(p,c),(c,r),(c,p),(c,c)\}$ contains **all possible combinations** of $A = \{r,p,c\}$ and $B = \{r,p,c\}$

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Notation: $A \times B$.

$$
\begin{array}{cc|c|c|c|}
\hline
 & r & (r,r) & (p,r) & (c,r) \\
\hline
\text{Set } B & p & (r,p) & (p,p) & (c,p) \\
\hline
 & c & (r,c) & (p,c) & (c,c) \\
\hline
\multicolumn{2}{c}{} & \multicolumn{1}{c}{r} & \multicolumn{1}{c}{p} & \multicolumn{1}{c}{c} \\
\multicolumn{2}{c}{} & \multicolumn{3}{c}{\text{Set } A}
\end{array}
$$

Cartesian Product

You've seen Cartesian products before:

	r	(r, r)	(p, r)	(c, r)
Set B	p	(r, p)	(p, p)	(c, p)
	c	(r, c)	(p, c)	(c, c)
		r	p	c
			Set A	

Cartesian Product

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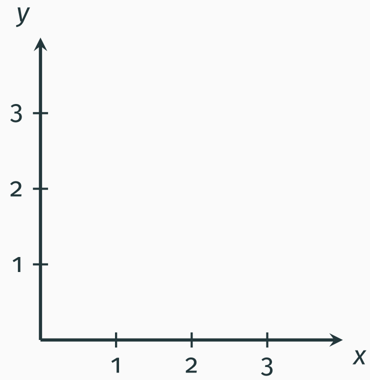

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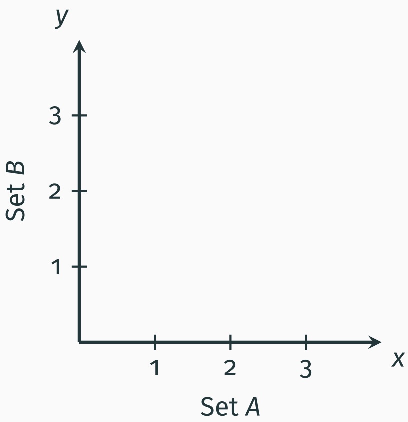

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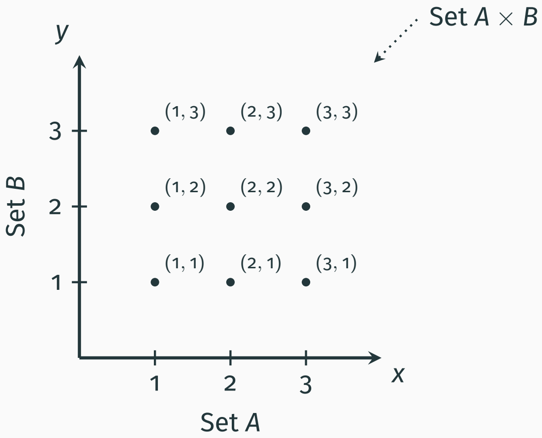


Indexed Sets

Sets may be numbered ("indexed"). Example:

Player 1's strategies: $S_1 = \{a, b\}$

Player 2's strategies: $S_2 = \{c, d\}$

Player 3's strategies: $S_3 = \{x, y\}$

We can define $S = S_1 \times S_2 \times S_3$

We can do this for any number n sets: $S = S_1 \times S_2 \times \cdots \times S_n$

Shorthand notation: $S = \times_{i=1}^{n} S_i$

Lists

Suppose $S_1 = \{r, p, c\}$ and $S_2 = \{r, p, c\}$.
So $S_1 \times S_2 = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$.

Elements of this set are **lists**.
Other names: **ordered pairs** (if 2), ***n*-tuples** (if n), **vectors**, **profiles**
List (r, c) means P1 played Rock and P2 played Scissors. Order matters!!

Sets vs. Lists:

Sets	**Lists**
Ex: $\{a, b, c, d\}$	Ex: (a, d, d, a)
Braces $\{\cdots\}$	Parentheses $(\cdots)$
Comma-separated	Comma-separated
Order doesn't matter	Order matters!!!
Cannot repeat	Can repeat

Confusing: $(0, 10)$ can represent a list or an interval. Context should clarify.

Cartesian Products & Lists

Rock-Paper-Scissors:
$S_1 = \{r, p, c\}$
$S_2 = \{r, p, c\}$
$S = S_1 \times S_2$ (Cartesian product)
$S = \{(r, r), (r, p), (r, c), (p, r), (p, p), (p, c), (c, r), (c, p), (c, c)\}$
Each element of S is a **list**

List says (1) what P1 chose in S_1, (2) what P2 chose in S_2

In game theory, lists of strategies are **strategy profiles**. e.g. (r, c).

Strategy Profiles With n Players

Game theory notation:

- Suppose a game has n players
- What does this mean: "Each player i chooses some $s_i \in S_i$"
- Any particular player number: $i \in \{1, 2, \ldots, n\}$
- Set of possible strategies for player i: S_i
- A particular strategy for player i: s_i

Player 1 chooses $s_1 \in S_1$
Player 2 chooses $s_2 \in S_2$
$\vdots$
Player n chooses $s_n \in S_n$

Strategy profile $s = (s_1, s_2, \ldots, s_n) \in S = S_1 \times S_2 \times \cdots S_n$

“Not i” Notation: s_{-i}

- Suppose there are n players
- Each player i plays strategy s_i
- The profile is $s = (s_1, s_2, s_3, \ldots, s_n)$
- Suppose we want to focus on player 7
  - “I am player 7”, so set $i = 7$
- In this case, $i = 7$. So $s_i = s_7$
- The strategies of others are $s_{-7} = (s_1, s_2, \ldots, s_6, s_8, \ldots, s_n)$
- For any player i, call this s_{-i}
  - $s_{-7} = (s_1, s_2, \ldots, s_6, s_8, \ldots, s_n)$
  - $s_{-3} = (s_1, s_2, s_4, \ldots, s_n)$
  - $s_{-1} = (s_2, s_3, \ldots, s_n)$
- The entire profile can be written as $s = (s_i, s_{-i})$ for any arbitrary i
  - s_i is what player i plays
  - s_{-i} is what all the other players play

Sets Can Be Functions

“Set-builder notation:”

- Suppose $X = \{1, 3, 5, 7, 9\}$. Define:

$$A = \{x \in X : x < 6\}$$

- Read as: “x in X such that $x < 6$”
- Thus, $A = \{1, 3, 5\}$

Sets can be functions of some parameter

- Again, suppose $X = \{1, 3, 5, 7, 9\}$. Define the set function:

$$A(b) = \{x \in X : x < b\}$$

- Here, b can vary (it is a *parameter* or *input*)
- $A(20) =$ $\qquad$ $A(8) =$
- $A(2) =$ $\qquad$ $A(0) =$

Game Theory Examples

- $\pi_i(s_i, s_{-i})$ will denote the payoff to player i at strategy profile $s = (s_i, s_{-i})$

		Player 2: U	Player 2: D
Player 1	L	3, 4	0, 5
	R	5, 0	0, 2

$\pi_1(U, L) = 3$, $\pi_1(U, R) = 0$, $\pi_2(D, R) = 2$, etc.

- Set of best replies for player i (payoff maximizers) against strategy profile s_{-i} is

$$BR_i(s_{-i}) = \{s_i \in S_i : \pi_i(s_i, s_{-i}) \geq \pi_i(a, s_{-i})\ \forall a \in S_i\}$$

- $BR_1(L) = \{D\}$ $\qquad$ $BR_1(R) = \{U, D\}$

Functions

Function you're probably used to: $y = x^2$ or $f(x) = x^2$

What is this function doing? Mapping one set into another!

Domain vs. Range (or "Codomain")

More General Functions

Any domain and any range. Ex: classes of animals.

Domain

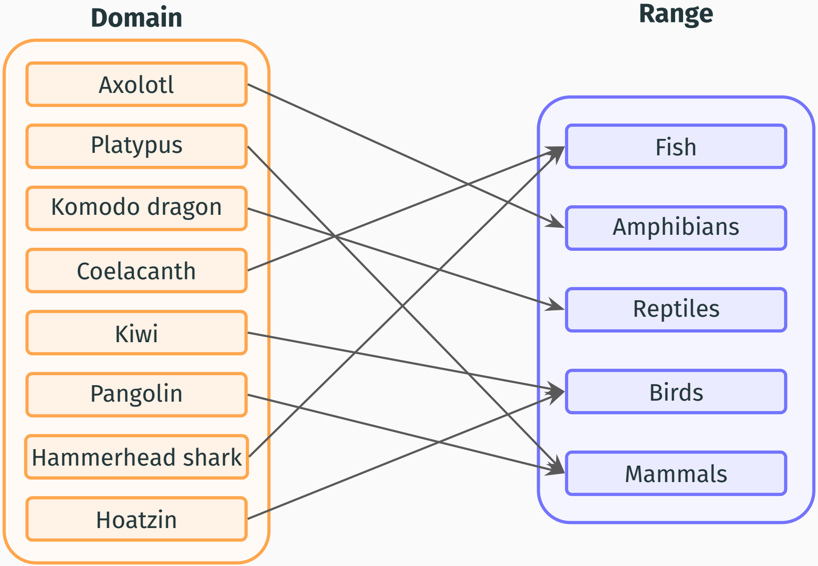
- Axolotl
- Platypus
- Komodo dragon
- Coelacanth
- Kiwi
- Pangolin
- Hammerhead shark
- Hoatzin

Range

- Fish
- Amphibians
- Reptiles
- Birds
- Mammals

More General Functions

Any domain and any range. Ex: classes of animals.



A Game Theory Example

Rock-Paper-Scissors:

9 possible strategy pairs: $S = \left\{ \begin{array}{ccc} \text{(r,r),} & \text{(r,p),} & \text{(r,c),} \\ \text{(p,r),} & \text{(p,p),} & \text{(p,c),} \\ \text{(c,r),} & \text{(c,p),} & \text{(c,c)} \end{array} \right\}$

First letter: P1's choice. Second letter: P2's choice

Outcomes: $O = \{\text{P1Wins}, \text{Tie}, \text{P2Wins}\}$

"Outcome function"

A Game Theory Example

We can represent this in table format.
Convention: Player 1 is rows, Player 2 is columns

Strategy pairs:

Set S_2

Set S_1		r	p	c
	r	(r,r)	(r,p)	(r,c)
	p	(p,r)	(p,p)	(p,c)
	c	(c,r)	(c,p)	(c,c)

$\rightarrow$

Outcome function:

	r	p	c
r			
p			
c			

A Game Theory Example

We can represent this in table format.
Convention: Player 1 is rows, Player 2 is columns

Strategy pairs:

Set S_2, Set S_1

	r	p	c
r	(r, r)	(r, p)	(r, c)
p	(p, r)	(p, p)	(p, c)
c	(c, r)	(c, p)	(c, c)

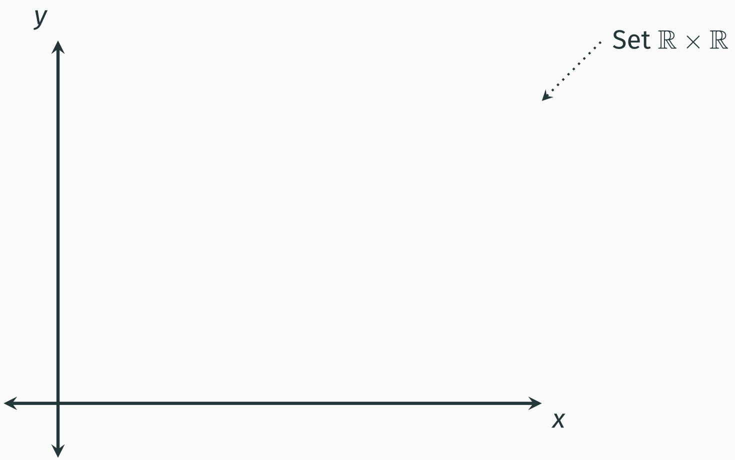
$\rightarrow$

Outcome function:

	r	p	c
r	Tie	P2	P1
p	P1	Tie	P2
c	P2	P1	Tie

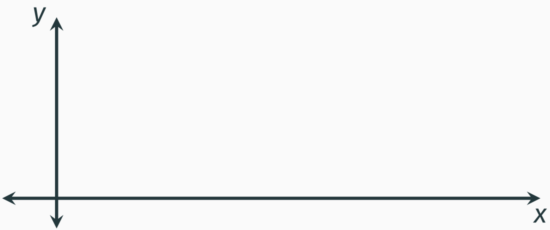
A Bit of Calculus

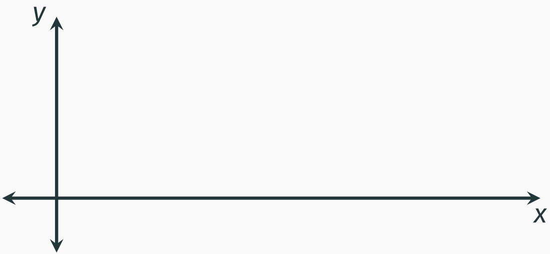
Set of all "real" numbers: $\mathbb{R} = (-\infty, \infty)$ (interval)
"Real" function: domain and range are both $\mathbb{R}$



A Bit of Calculus

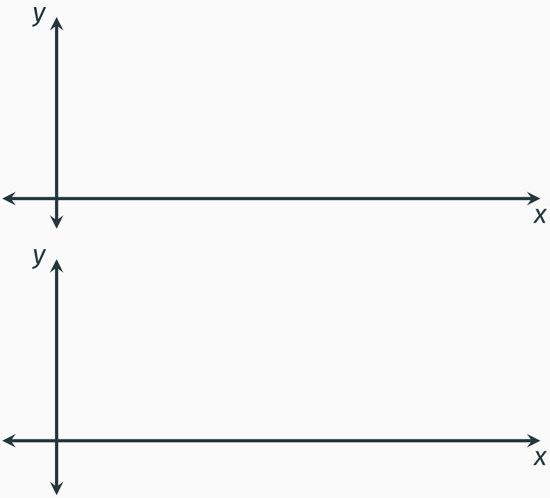
What is the slope at a point?





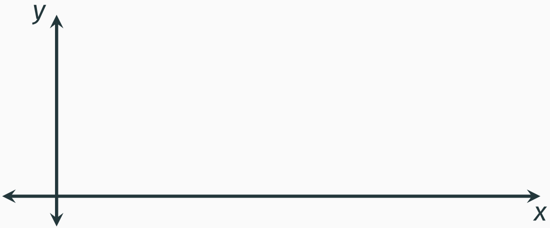
A Bit of Calculus

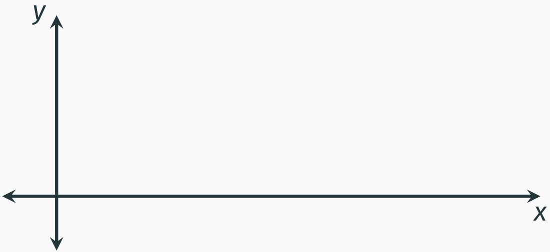
What is the derivative?



A Bit of Calculus

How to find the max of a function using calculus?

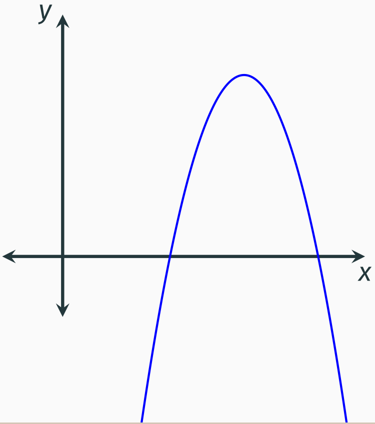




A Bit of Calculus

A worked example.

- Function: $f(x) = -4 + 12x - 2x^2$
- Derivative: $f'(x) =$
- Set equal to zero:
- Solve:



The End!

That's enough for now!

Please ask questions if ever some math notation is unclear.